program e503 prescott
#This program is based on the model described in "On the Needed Quantity of Government Debt"
#Federal Reserve Bank of Minneapolis Working Paper 648 by Kathryn Birkeland and Edward Prescott
#Modified by B. Goff for classroom use as example of simple DSGE framework using Gretl
#Modifactions produce output growth (Y) that approximates post WWII average growth and variablility

#Defining variables

#Inputs:	AR	=	retirement age less 20 years
#			(note: the agent is born at t=0 and age=20)
#	AD	=	lifespan less 20 years
#	gamma	=	growth rate of technology
#	eta	=	growth rate of the population
#	irate	=	interest rate
#	theta	=	capital share in the cobb-douglas production function#
	delta	=	depreciation rate of the capital stock
#	alpha	=	preference parameter
#	Z	=	labor-augmenting technology in production # #
#	beta	=	time discount factor of the household
#	Npop	=	size of the initial population
#	tauh	=	input labor income tax rate
#	tauk	=	input tax rate on net capital income
#	g	=	growth rate of aggregate output
#	Ncohort	=	size of the cohort entering the workforce in period 1
#	Nwork	=	size of the working population in period 1
#	r	=	rental price of capital
#	HK	=	aggregate capital labor ratio
#	w	=	wage rate
#	CH	=	aggregate consumption to labor ratio
#	cw	=	consumption of an individual
#	hi	=	individual labor supply
#	H	=	aggregate labor supply
#	C	=	aggregate consumption
#	K	=	aggregate capital stock
#	Y	=	aggregate output
#	X	=	aggregate investment
#	psi	=	individual transfer from the government
#	A	=	assets of an individual born in period ###
#	TotA	=	aggregate assets
#	rev	=	total tax revenue
#	trans	=	total lump-sum transfers
#	D	=	government debt
#	singleA	=	assets of an individual in each cohort
#	utility	=	lifetime utility of a person entering the workforce in t=1
#			utility/discount can be used for the welfare comparison
#			in terms of lifetime consumption equivalents

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#Create data set with 100 years of observations
nulldata 100

#Define & calibrate model parameters
   series e = normal()
   series gamma = 0.02 + .005*e

         scalar eta=0.03
         scalar irate = 0.04
         scalar theta = 0.35
         scalar delta = 0.05
         scalar alpha = 2.14626
         scalar Z = 0.026888
         scalar beta = 0.980768

         scalar tauh = 0.4
         scalar tauk = 0.2

         scalar dep = 0.175
         scalar ARet = 45
         scalar AD = 65
         series t = time

#Equations defining the economy start here

#growth rate of per capita consumption
genr g = (1+gamma)*(1+eta)-1


smpl 1 1
series Npop = 100
smpl 2 100
series Npop = (1+eta)*Npop(-1)

smpl 1 100

#Compute the size of the cohort born in period one given the initial population

series xeta = 0
  loop i=1..AD
  series xeta = xeta + (1/(1+eta))^i
  endloop

#This makes cohort equal to the pop (a simplification of Prescott)
genr Ncohort   = Npop
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#Find the size of the working population given the size of cohort one
series xwork=0
  loop i=1..ARet
   series xwork = xwork + (1/(1+eta))^i
   endloop

genr Nwork = Npop

#Compute the rental rate on capital, capital labor ratio, and wage
genr r = irate/(1-tauk)+delta
genr HK    = ((r/theta)^(1/(1-theta)))/Z
genr w = Z^(1-theta)*(1-theta)*(HK)^(-theta)
genr KH         = 1/HK

#Compute individual consumption and labor
genr CH         = Z^(1-theta)*KH^theta - (g+delta)*KH
genr cw = 1/(alpha/((1-tauh)*w) + Npop/(CH*Nwork))
genr hi = 1 - alpha*cw/(w*(1-tauh))

#Compute aggregate labor, consumption, capital, output and investment
genr H = Nwork*hi
genr Ci = Npop*cw
genr K = KH*H
genr Y= (K^theta)*(Z*H)^(1-theta)
          genr X = Y-Ci
genr Ypct = 100*ldiff(Y)



#The following section adds government sector to model
#Compute the government transfer from the lifetime budget constraint
# and Discount term for labor income
series xr = 0
  loop i= 1..ARet
   series xr = xr + ((1+gamma)/(1+irate))^(i-1)
  endloop

#Compute Discount term for consumption
series xd = 0
  loop i=1..AD
  series  xd = xd + ((1+gamma)/(1+irate))^(i-1)
  endloop

#Compute Transfer at individual level
series psi = cw - (1-tauh)*w*hi*xr/xd
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#Compute AD+1 and ARet+1 for following loop
genr  ADp1 = AD + 1
genr ARetp1 = ARet + 1

genr ADp2 = ADp1 + 1

#Compute the lifetime asset profile of a person in cohort one
#Start by Initialize beginning period assets
 smpl 1 1
 series A= 0
 smpl 2 66

#Then Generate assets over rest of life before and after retirement
    loop i=2..ADp1
      series A = (i<=ARetp1) ? A(-1)*(1+irate)+((1-tauh)*hi*w+psi-cw)*(1+gamma)^(i-1) : A(-
1)*(1+irate)+(psi-cw)*(1+gamma)^(i-1)
     endloop


#Compute the value of total assets in the economy
smpl 1 66
series TotA = 0.0
series workA = 0

smpl 2 66
    loop i=2..ADp1
    series workA= A*Ncohort/((1+eta)^i*(1+gamma)^i)
    endloop

smpl 1 66
series TotA = TotA + workA

#Compute Government Accounts
series rev = tauh*w*H + tauk*(r-delta)*K
series trans      = Npop*psi
series Debtg = TotA - K
series debt2      = (rev-trans)/(irate-g)

#Compute household and government balance sheet items
series HHK = (Debtg <=0) ? TotA : K
series HHD = (Debtg <=0) ?  0 : Debtg
series GovK = (Debtg <=0) ? -Debtg : 0
series GovD = (Debtg<=0) ? 0 : Debtg
series discount = 0.0
  loop i=1..AD
  series dis = beta^i
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  series discount = discount + dis
  endloop

series discounth = 0.0
  loop i=1..ARet
  series dish = beta^i
  series discounth = discounth + dish
   endloop

series utility = log(cw)*discount + alpha*log(1-hi)*discounth

scalar debtequity=0.8
genr debtpriv = TotA*debtequity
genr debty = debtpriv/Y
   genr ipayments = (1+irate)*debtpriv
                    genr difyipayments= Y - ipayments


#Compute Assets of cohort 1
  loop i=1..ADp1
  series singleA= A/((1+g)^i)
  endloop

#Create time series structure for dataset using 1970 as beginning year
setobs 1 1970 --time-series
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